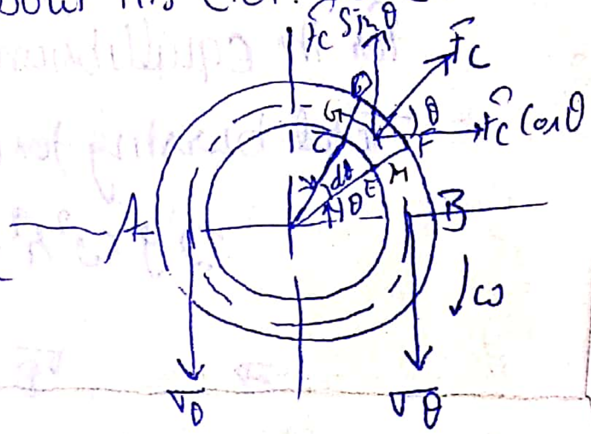
Rotating Rims & Discs ①

There are many machine members which are of rotating type. Due to rotation centrifugal stresses are introduced in these members. So in this unit, we have to calculate stresses induced due to rotation. It shall be assumed that the density of material is uniform throughout.

STRESSES IN ROTATING RINGS :-

Consider a thin rotating ring about its C.G. at O.



Let ρ = Density of ring in kg/m^3

r = Mean radius of ring in m.

ω = Angular speed of rotation in rad/s.

t = Thickness of ring, m.

Consider an element of ring CDFE between θ & $\theta + d\theta$. Then

$$GH = r\,d\theta$$

Volume of element per unit length = $r\,d\theta \cdot t$

Centrifugal force on this element

$$dF_c = \rho\, r\,d\theta \cdot t\, \omega^2 r$$

Vertical component of this force = $dF_c \sin\theta$

$$= \rho\, r\,d\theta\, t\,\omega^2 r \sin\theta$$

Total vertical or bursting force across diameter AB becomes

$$\int_0^{\pi} dF_c \sin\theta = \int_0^{\pi} \rho\, r\,d\theta\, t\,\omega^2 r \sin\theta$$

$$= \rho\omega^2 r^2 t \int_0^{\pi} \sin\theta \, d\theta$$

$$= \rho\omega^2 r^2 t \left| -\cos\theta \right|_0^{\pi}$$

$$= \rho\omega^2 r^2 t \left\{ -(\cos\pi + \cos 0) \right\}$$

$$= 2\rho\omega^2 r^2 t$$

If σ_θ = Hoop stress induced in the ring in N/m^2

Then the total resisting force becomes $= 2\sigma_\theta \cdot t \cdot 1$

For the equillibrium of the ring

Total bursting force = Total resisting force.

$$2\rho\omega^2 r^2 t = 2\sigma_\theta t$$

$$\Rightarrow \quad \sigma_\theta = \rho\omega^2 r^2$$

$$V = r\omega$$

$$\therefore \boxed{\sigma_\theta = \rho V^2}$$

Stress in Rotating Disc :-

Consider a flat disc of uniform thickness t. Let r_1 & r_2 be the inner & outer radii of the disc rotating at speed ω.

An element of Disc ABCD at radius r is acted upon by stress σ_r & $\sigma_r + d\sigma_r$ on faces AD & BC respectively, & by stress σ_θ on the faces AB & CD. On the flat faces of disc, there is no normal stress

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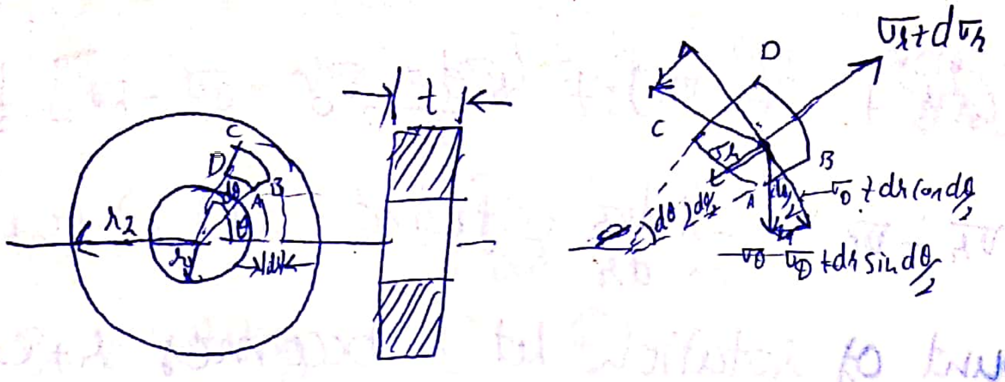

Volume of element ABCD $= r\,d\theta . dr\, t$

Radial force on the element ABCD due to rotation

$= \rho\, r\, d\theta . dr . t\, \omega^2 r$

Outward force on the face BC $= (r+dr)\,d\theta . t\,(\sigma_r + d\sigma_r)$

Inward force on the face AD, $= r\,d\theta . t\, \sigma_r$

Forces on the face AB & DC $= \sigma_\theta . t . dr$

Horizontal components $= 2[\sigma_\theta\, t\, dr \sin d\theta/2]$

If $d\theta$ is small

then $\sin d\theta/2 \rightarrow d\theta/2$

$\therefore$ forces are $= \cancel{2}\,\sigma_\theta\, t\, dr\, \frac{d\theta}{\cancel{2}}$

$= \sigma_\theta\, t\, dr\, d\theta$

$\therefore$ Resolving forces in the radial outward direction, we get

$(\sigma_r + d\sigma_r)(r+dr)\,d\theta . t - \sigma_r . r\,d\theta . t - \sigma_\theta\, t\, dr . d\theta + \rho\, r\, d\theta\, dr\, t\, \omega^2 r = 0$

~~$\rho\, r\, d\theta . dr\, t\, \omega^2 r + (\sigma_r - \sigma_\theta)\, dr\, d\theta . t + d\sigma_r\, r\, d\theta\, t = 0$~~

$\therefore \rho\, \omega^2 r^2\, t\, d\theta\, dr + \sigma_r \cancel{r\,d\theta}\, t + \sigma_r\, dr\, d\theta\, t + d\sigma_r\, r\, d\theta\, t$

$+ \underbrace{d\sigma_r\, dr\, d\theta\, t}_{\text{neglecting}} - \sigma_r\, \cancel{r\,d\theta}\, t - \sigma_\theta\, t\, dr\, d\theta = 0$

$\rho\, \omega^2 r^2\, \cancel{t\, d\theta}\, dr + \sigma_r\, dr\, \cancel{d\theta\, t} + d\sigma_r . r\, \cancel{d\theta\, t}$

$- \sigma_\theta\, \cancel{t}\, dr\, \cancel{d\theta} = 0$

$$\therefore \rho\omega^2 r^2 + (\sigma_r - \sigma_\theta) + r\frac{d\sigma_r}{dr} = 0$$

$$\Rightarrow \sigma_r - \sigma_\theta = -r\frac{d\sigma_r}{dr} - \rho\omega^2 r^2 \quad (1)$$

On account of rotation let r becomes $r+u$ & $r+dr$ becomes $r+dr+du$

$\therefore$ Circumferential strain

$$\epsilon_\theta = \frac{2\pi(r+u) - 2\pi r}{2\pi r} = \frac{u}{r}$$

Also $$\epsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu\sigma_r]$$

$$\therefore \frac{u}{r} = \frac{1}{E}[\sigma_\theta - \nu\sigma_r]$$

$$\Rightarrow u = \frac{r}{E}[\sigma_\theta - \nu\sigma_r]$$

By Diff. w.r.t. r.

$$\frac{du}{dr} = \frac{1}{E}[\sigma_\theta - \nu\sigma_r] + \frac{r}{E}\left[\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right] \quad (2)$$

Now Radial strain

$$\epsilon_r = \frac{(r+dr+du) - (r+dr)}{dr} = \frac{du}{dr}$$

Also $$\epsilon_r = \frac{1}{E}[\sigma_r - \nu\sigma_\theta]$$

$$\therefore \frac{du}{dr} = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) \quad (3)$$

By comparing (2) & (3)

$$\therefore \frac{1}{E}[\sigma_\theta - \nu\sigma_r] + \frac{r}{E}\left[\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right] = \frac{1}{E}(\sigma_r - \nu\sigma_\theta)$$

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$$\Rightarrow \frac{1}{E}\left[\sigma_r - \sigma_\theta - \nu\sigma_\theta + \nu\sigma_r\right] = \frac{r}{E}\left[\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right]$$

$$\Rightarrow (1+\nu)(\sigma_r - \sigma_\theta) = r\left[\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right] \quad \text{—— } 4$$

By substituting eqn 1 in 4

$$(1+\nu)\left(-r\frac{d\sigma_r}{dr} - \rho\omega^2 r^2\right) = r\left(\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right)$$

Simplifying we get

$$\frac{d(\sigma_r + \sigma_\theta)}{dr} + (1+\nu)\rho\omega^2 r = 0$$

By Integrating,

$$\sigma_r + \sigma_\theta + (1+\nu)\frac{\rho\omega^2 r^2}{2} = C_1 \quad \text{—— } 5$$

$$\therefore \sigma_\theta = C_1 - \sigma_r - \tfrac{1}{2}(1+\nu)\rho\omega^2 r^2$$

By putting σ_θ in eqn 1

$$2\sigma_r + r\frac{d\sigma_r}{dr} = C_1 - \frac{(3+\nu)}{2}\rho\omega^2 r^2$$

Multiplying both sides by r

$$2r\sigma_r + r^2\frac{d\sigma_r}{dr} = C_1 r - \frac{3+\nu}{2}\rho\omega^2 r^3$$

~~By integrating~~

$$\frac{d(r^2\sigma_r)}{dr} = C_1 r - \frac{(3+\nu)}{2}\rho\omega^2 r^3$$

By integrating

$$r^2\sigma_r = C_1\frac{r^2}{2} - \frac{(3+\nu)}{2}\rho\omega^2\frac{r^4}{4} + C_2$$

$$\therefore \boxed{\sigma_r = \frac{C_1}{2} - \frac{(3+\nu)}{8}\rho\omega^2 r^2 + \frac{C_2}{r^2}} \quad \text{—— } 6$$

By putting σ_r in eqn 5

$$\therefore \boxed{\sigma_\theta = \frac{C_1}{2} - \frac{C_2}{r^2} - \left(\frac{1+3\nu}{8}\right)\rho\,\omega^2 r^2} \quad (7)$$

$\therefore$ Radial Displacement

$$u = \frac{r}{E}\left[\sigma_\theta - \nu\sigma_r\right]$$

$$\boxed{u = \frac{r}{E}\left[(1-\nu)\frac{C_1}{2} - (1+\nu)\frac{C_2}{r^2} - \left(\frac{1-\nu^2}{8}\right)\rho\omega^2 r^2\right]}$$

{$\because$ By putting σ_r & σ_θ from eqn (6) & (7)}

Case I When Disc is solid

(a) The boundary conditions are

At $r=0$, $u=0$

& $r=r_2$, $\sigma_r=0$

The general equation of σ_r & σ_θ are

$$\sigma_r = \frac{C_1}{2} + \frac{C_2}{r^2} - \left(\frac{3+\nu}{8}\right)\rho\omega^2 r^2$$

$$\sigma_\theta = \frac{C_1}{2} - \frac{C_2}{r^2} - \left(\frac{1+3\nu}{8}\right)\rho\omega^2 r^2$$

At $r=0$, $u=0$ & σ_r & σ_θ becomes infinite from the above equations, which is not possible

Hence $C_2=0$

$$\therefore \sigma_r = \frac{C_1}{2} - \frac{(3+\nu)}{8}\rho\omega^2 r^2$$

$$\& \ \sigma_\theta = \frac{C_1}{2} - \left(\frac{1+3\nu}{8}\right)\rho\omega^2 r^2$$

At $r = r_2$ $\sigma_r = 0$

$$\therefore C_1 = \left(\frac{3+\nu}{4}\right) \rho \omega^2 r_2^2$$

$$\therefore \boxed{\sigma_r = \frac{(3+\nu)}{8} \rho \omega^2 (r_2^2 - r^2)} \text{ — } (a)$$

$$\& \quad \sigma_\theta = \frac{(3+\nu)}{8} \rho \omega^2 r_2^2 - \frac{(1+3\nu)}{8} \rho \omega^2 r^2$$

$$\text{Or} \quad \boxed{\sigma_\theta = \frac{\rho}{8} \omega^2 \left[(3+\nu) r_2^2 - (1+3\nu) r^2 \right]} \text{ — } (b)$$

At $r = r_2$; $\sigma_\theta = \frac{(1-\nu)}{4} \rho \omega^2 r_2^2$

The value of σ_r & σ_θ are max. at $r = 0$

$$\therefore \boxed{(\sigma_r)_{max} = \left(\frac{3+\nu}{8}\right) \rho \omega^2 r_2^2} \text{ — } (c)$$

$$\boxed{\& \ (\sigma_\theta)_{max} = \left(\frac{3+\nu}{8}\right) \rho \omega^2 r_2^2} \text{ — } (d)$$

Radial displacement of disc becomes

$$u = \frac{r}{E} [\sigma_\theta - \nu \sigma_r]$$

$$= \frac{r}{E} \left[\left\{ \left(\frac{3+\nu}{8}\right) \rho \omega^2 r_2^2 - \left(\frac{1+3\nu}{8}\right) \rho \omega^2 r^2 \right\} \right] - \nu \left[\left(\frac{3+\nu}{8}\right) \rho \omega^2 (r_2^2 - r^2) \right]$$

$$\boxed{u = \frac{r(1-\nu)}{8E} \rho \omega^2 \left[(3+\nu) r_2^2 - (1+\nu) r^2 \right]}$$

At $r = r_2$

$$\boxed{u = \frac{1-\nu}{4E} \times \rho \omega^2 r_2^3}$$

Hollow Disc :-

$$\sigma_r = \frac{C_1}{2} + \frac{C_2}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2$$

& $$\sigma_\theta = \frac{C_1}{2} - \frac{C_2}{r^2} - \left(\frac{1+3\nu}{8}\right)\rho\omega^2 r^2$$

At $r = r_1$ $\sigma_r = 0$

& $r = r_2$ $\sigma_r = 0$

$$\therefore 0 = \frac{C_1}{2} + \frac{C_2}{r_1^2} - \left(\frac{3+\nu}{8}\right)\rho\omega^2 r_1^2$$

$$0 = \frac{C_1}{2} - \frac{C_2}{r_2^2} - \left(\frac{3+\nu}{8}\right)\rho\omega^2 r_2^2$$

$$\therefore C_1 = \left(\frac{3+\nu}{4}\right)\rho\omega^2 (r_1^2 + r_2^2)$$

& $$C_2 = -\frac{(3+\nu)}{4}\rho\omega^2 r_1^2 r_2^2$$

$$\therefore \sigma_r = \left(\frac{3+\nu}{8}\right)\rho\omega^2\left[r_1^2 + r_2^2 - r^2 - \frac{r_1^2 r_2^2}{r^2}\right]$$

& $$\sigma_\theta = \left(\frac{3+\nu}{8}\right)\rho\omega^2\left[r_1^2 + r_2^2 + \frac{r_1^2 r_2^2}{r^2} - \left(\frac{1+3\nu}{3+\nu}\right)r^2\right]$$

σ_θ is max, $r = r_1$

$$\boxed{(\sigma_\theta)_{max} = \left(\frac{3+\nu}{4}\right)\rho\omega^2\left[r_2^2 + \left(\frac{1-\nu}{3+\nu}\right)r_1^2\right]}$$

for σ_r to be max.

$$\frac{d\sigma_r}{dr} = 0$$

$$2r - \frac{2r_1^2 r_2^2}{r^3} = 0$$

$$\Rightarrow \boxed{r = \sqrt{r_1 r_2}}$$

$$\therefore \boxed{(\sigma_r)_{max} = \left(\frac{3+\nu}{8}\right)\rho\omega^2 (r_2 - r_1)^2}$$

When $r_1 \to 0$ $$\boxed{(\sigma_\theta)_{max} = \left(\frac{3+\nu}{\cancel{8}\,4}\right)\rho\omega^2 r_2^2}$$

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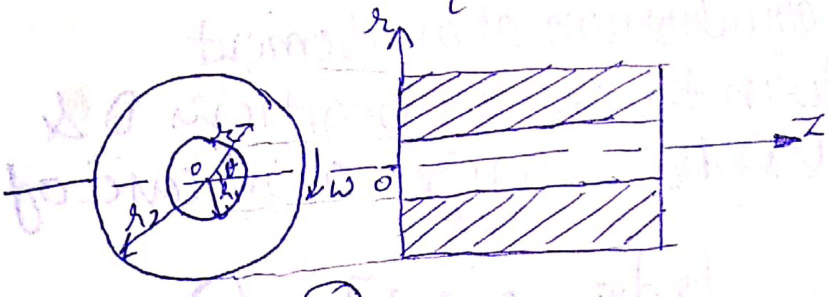
Stresses in Rotating Cylinder:-

Consider a circular cylinder of inside radius r_1 & outside radius r_2 rotating at speed ω.
Assume that plane sections of cylinder remain plane during rotation, then the axial strain along the z-axis will be independent of radius r of the cylinder & will be constant.

Radial strain $\varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)] = \frac{du}{dr}$ — (a)

Hoop strain $\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] = u/r$ — (b)

Axial strain $\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r + \sigma_\theta)]$ — (c)



From eqn (b)

$$E u = r[\sigma_\theta - \nu(\sigma_r + \sigma_z)]$$

Diff. w.r.t r, we get

$$E\frac{du}{dr} = [\sigma_\theta - \nu(\sigma_r + \sigma_z)] + r\left[\frac{d\sigma_\theta}{dr} - \nu\left(\frac{d\sigma_r}{dr} + \frac{d\sigma_z}{dr}\right)\right]$$

By putting the value of $\frac{du}{dr}$ from eqn (a)

$$E\frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)] = [\sigma_\theta - \nu(\sigma_r + \sigma_z)] + r\left[\frac{d\sigma_\theta}{dr} - \nu\left(\frac{d\sigma_r}{dr} + \frac{d\sigma_z}{dr}\right)\right]$$

$$\therefore (\sigma_r - \sigma_\theta)(1+\nu) = r\left[\frac{d\sigma_\theta}{dr} - \nu\left(\frac{d\sigma_r}{dr} + \frac{d\sigma_z}{dr}\right)\right] \quad \text{—(d)}$$

From eqn (c) we get

$$E\varepsilon_z = \sigma_z - \nu(\sigma_r + \sigma_\theta) = \text{Constant} = C_1$$

$$\therefore \sigma_z = C_1 + \nu(\sigma_r + \sigma_\theta)$$

Diff. w.r.t r, we get

$$\frac{d\sigma_z}{dr} = \nu\left(\frac{d\sigma_r}{dr} + \frac{d\sigma_\theta}{dr}\right)$$

By substituting $\frac{d\sigma_z}{dr}$ in eqn (d)

$$\therefore (\sigma_r - \sigma_\theta)(1+\nu) = r\left[\frac{d\sigma_\theta}{dr} - \nu\left\{\frac{d\sigma_r}{dr} + \nu\left(\frac{d\sigma_r}{dr} + \frac{d\sigma_\theta}{dr}\right)\right\}\right]$$

$$= r\left[\frac{d\sigma_\theta}{dr}(1-\nu^2) - \nu(1+\nu)\frac{d\sigma_r}{dr}\right]$$

$$(\sigma_r - \sigma_\theta)\cancel{(1+\nu)} = \cancel{(1+\nu)}\, r\left[(1-\nu)\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right]$$

$$\sigma_r - \sigma_\theta = r\left[(1-\nu)\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right] \quad (e)$$

Also considering the equillibrium of an element of the cylinder between the angular positions θ & $\theta + d\theta$ & radii r & $r+dr$, we can get as in case of rotating disc

$$\sigma_r - \sigma_\theta = -\left[r\frac{d\sigma_r}{dr} + \rho\omega^2 r^2\right] \quad (f)$$

By comparing the eqns (e) & (f)

$$r\left[(1-\nu)\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right] = -\left[r\frac{d\sigma_r}{dr} + \rho\omega^2 r^2\right]$$

$$\Rightarrow (1-\nu)\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr} = -\left(\frac{d\sigma_r}{dr} + \rho\omega^2 r\right)$$

$$\Rightarrow (1-\nu)\frac{d\sigma_r}{dr} + (1-\nu)\frac{d\sigma_\theta}{dr} = -\rho\omega^2 r$$

$$\Rightarrow (1-\nu)\frac{d(\sigma_r + \sigma_\theta)}{dr} = -\rho\omega^2 r$$

$$\Rightarrow \frac{d(\sigma_r + \sigma_\theta)}{dr} = -\frac{\rho}{1-\nu}\omega^2 r$$

By integrating both sides w.r.t r

$$\sigma_r + \sigma_\theta = -\frac{\rho}{1-\nu}\omega^2\frac{r^2}{2} + C_2 \quad \text{—(g)}$$

where C_2 is const. of integration

By adding eqns (f) & (g)

$$2\sigma_r = -\left[r\frac{d\sigma_r}{dr} + \rho\omega^2 r^2\right] - \frac{\rho}{1-\nu}\frac{\omega^2 r^2}{2} + C_2$$

$$2\sigma_r + r\frac{d\sigma_r}{dr} = -\rho\omega^2 r^2\left[\frac{3-2\nu}{2(1-\nu)}\right] + C_2$$

Multiplying both sides by r

$$2r\sigma_r + r^2\frac{d\sigma_r}{dr} = -\rho\frac{\omega^2 r^3}{2}\left[\frac{3-2\nu}{(1-\nu)}\right] + rC_2$$

$$\Rightarrow \frac{d(r^2\sigma_r)}{dr} = -\rho\frac{\omega^2 r^3}{2}\left[\frac{3-2\nu}{1-\nu}\right] + rC_2$$

By Integration

$$r^2\sigma_r = -\rho\omega^2\frac{r^4}{2\times4}\left(\frac{3-2\nu}{1-\nu}\right) + \frac{r^2}{2}C_2 + C_3$$

$$\therefore \sigma_r = -\rho\frac{\omega^2 r^2}{8}\left(\frac{3-2\nu}{1-\nu}\right) + \frac{\cancel{r^2}}{\cancel{r^2}\,2}C_2 + \frac{C_3}{r^2}$$

$$\Rightarrow \boxed{\sigma_r = -\frac{\rho\omega^2 r^2}{8}\left(\frac{3-2\nu}{1-\nu}\right) + \frac{C_2}{2} + \frac{C_3}{r^2}} \quad \text{—} \boxed{1}$$

By putting value of σ_r in eqn (g)

$$\therefore \boxed{\sigma_\theta = -\rho\frac{\omega^2 r^2}{8}\left(\frac{1+2\nu}{1-\nu}\right) + \frac{C_2}{2} - \frac{C_3}{r^2}} \quad \text{—} \boxed{2}$$

Eqn (1) & (2) are known as governing eqn for a rotating cylinder.

Solid Cylinder :- The equations for cylinder are

$$\sigma_r = \frac{C_2}{2} + \frac{C_3}{r^2} - \frac{\rho\omega^2 r^2}{8}\left(\frac{3+2\nu}{1-\nu}\right)$$

$$\sigma_\theta = \frac{C_2}{2} - \frac{C_3}{r^2} - \frac{\rho\omega^2 r^2}{8}\left(\frac{1+2\nu}{1-\nu}\right)$$

Constant C_3 must be zero, since the stress remain finite at $r=0$

$$\therefore \sigma_r = \frac{C_2}{2} - \frac{1}{8}\left(\frac{3+2\nu}{1-\nu}\right)\rho\omega^2 r^2$$

$$\& \quad \sigma_\theta = \frac{C_2}{2} - \frac{1}{8}\left(\frac{1+2\nu}{1-\nu}\right)\rho\omega^2 r^2$$

For a solid cylinder with a free surface, at $r=r_2$, $\sigma_r = 0$

$$\therefore \frac{C_2}{2} = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_2^2$$

$$\therefore \boxed{\sigma_r = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 (r_2^2 - r^2)} \quad (3)$$

$$\boxed{\& \quad \sigma_\theta = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2\left[r_2^2 - \left(\frac{1+2\nu}{3-2\nu}\right)r^2\right]} \quad (4)$$

The maximum stress occur at the centre of cylinder, where $r=0$

$$\boxed{(\sigma_r)_{max} = (\sigma_\theta)_{max} = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_2^2} \quad (5)$$

Hollow Cylinder :-

$$\sigma_r = \frac{C_2}{2} + \frac{C_3}{r^2} - \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r^2$$

The boundary conditions are 7

$\sigma_r = 0$ at $r = r_1$

$\sigma_r = 0$ at $r = r_2$

$$\therefore \quad 0 = \frac{C_2}{2} + \frac{C_3}{r_1^2} - \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_1^2 \quad (i)$$

$$0 = \frac{C_2}{2} + \frac{C_3}{r_2^2} - \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_2^2 \quad (ii)$$

By solving eqn (i) & eqn (ii).

$$\frac{C_2}{2} = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2\left(r_1^2 + r_2^2\right)$$

$$\& \quad C_3 = -\frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_1^2 r_2^2$$

$$\therefore \boxed{\sigma_r = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2\left[r_1^2 + r_2^2 - \frac{r_1^2 r_2^2}{r^2} - r^2\right]}$$

$$\& \boxed{\sigma_\theta = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2\left[r_1^2 + r_2^2 + \frac{r_1^2 r_2^2}{r^2} - \left(\frac{1+2\nu}{3-2\nu}\right)r^2\right]}$$

σ_θ is maximum at $r = r_1$

$$\therefore (\sigma_\theta)_{max} = \frac{1}{4}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_2^2\left[1 + \left(\frac{1-2\nu}{3-2\nu}\right)\frac{r_1^2}{r_2^2}\right]$$

If $\frac{r_1}{r_2} = 0$, then

$$\boxed{(\sigma_\theta)_{max} = \frac{1}{4}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2 r_2^2}$$

$\therefore$ Max. hoop stress in a cylinder with a small hole at the centre is twice that of in a solid cylinder

For σ_r to be max., $\frac{d\sigma_r}{dr} = 0$

$$-2r + \frac{2r_1^2 r_2^2}{r^3} = 0$$

$$r = \sqrt{r_1 r_2}$$

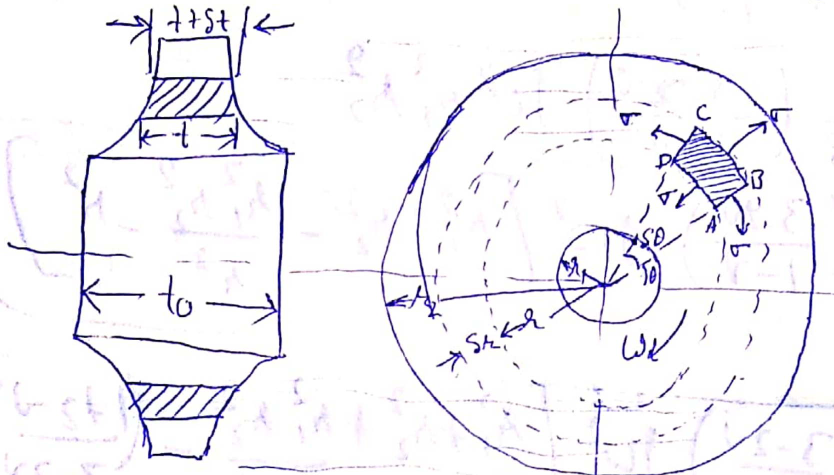
$$\therefore \boxed{(\sigma_r)_{max} = \frac{1}{8}\left(\frac{3-2\nu}{1-\nu}\right)\rho\omega^2\left(r_2 - r_1\right)^2}$$

Disc of Uniform Strength :-

If the disc is to have uniform strength, then the radial & hoop stresses must be equal at all points in the disc. Hence

$$\sigma_\theta = \sigma_r = \sigma = \text{Constant}$$

Consider the equillibrium of element ABCD of disc. Let t be the thickness of the radius at radius r & $t+\delta t$ at radius $r+\delta r$.



Outward radial force acting on face BC

$$= \sigma(t+\delta t)(r+\delta r)\,\delta\theta$$

$$= \sigma(tr + t\delta r + \delta t\, r + \delta t\, \delta r)\,\delta\theta$$

Centrifugal force on the element ABCD is

$$= \rho(r\delta\theta \cdot \delta r \cdot t)\,\omega^2 r$$

Inward radial force acting on face AD $= \sigma t r \cdot \delta\theta$

Inward radial force due to component of forces acting on faces AB & CD $= \sigma t\, \delta r\, \delta\theta$

For the equillibrium of the element

Total inward radial force = Total outward radial force

$$\sigma t r \delta\theta + \sigma t \delta r \delta\theta = \sigma(tr + r\delta t + t\delta r) + \rho(r\delta\theta . \delta r . t)\omega^2 r$$

$$\therefore \sigma . r . \delta t . \delta\theta + \rho \delta\theta . \delta r t \omega^2 r^2 = 0$$

$$\frac{\delta t}{t} = -\frac{\rho\omega^2}{\sigma} r . \delta r$$

$$\frac{dt}{t} = -\rho\frac{\omega^2}{\sigma} r\, dr$$

By integrating

$$\log t = -\rho\frac{\omega^2}{\sigma}\frac{r^2}{2} + \log A$$

where $\log A$ is constant of integration

$$\log \frac{t}{A} = -\rho\frac{\omega^2 r^2}{\sigma . 2}$$

$$\therefore t = A e^{-\rho\frac{\omega^2 r^2}{2\sigma}}$$

Let $t = t_0$ at $r = r_1$ then

$$t_0 = A e^{-\rho\left(\frac{\omega^2 r_1^2}{2\sigma}\right)}$$

$$\therefore A = t_0 e^{\rho\left(\frac{\omega^2 r_1^2}{2\sigma}\right)}$$

$$\therefore \boxed{t = t_0 e^{-\rho\frac{\omega^2}{2\sigma}(r^2 - r_1^2)}}$$

Which gives the thickness of disc at any radius

$$\text{Let } \lambda = \sqrt{\frac{2\sigma}{\rho\omega^2}}$$

$$\therefore \boxed{t = t_0 e^{-\left(\frac{r^2 - r_1^2}{\lambda^2}\right)}}$$

$$\frac{t}{t_0} = e^{-\left(\frac{r^2 - r_1^2}{\lambda^2}\right)}$$

$$\therefore \frac{d(t/t_0)}{d(r/\lambda)} = 2r/\lambda \cdot e^{-\left(\frac{r^2 - r_1^2}{\lambda^2}\right)} = -\frac{2r}{\lambda}\cdot\frac{t}{t_0}$$

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which is slope of curve. It is zero for $r/\lambda = 0$ & $\frac{t}{t_0} = 0$ occurring at $r/\lambda = \infty$

The curvature is

$$\frac{d^2(t/t_0)}{d(r/\lambda)^2} = -\frac{2t}{t_0} - \frac{2r}{\lambda}\frac{d(t/t_0)}{d(r/\lambda)}$$

$$= -\frac{2t}{t_0} - \frac{2r}{\lambda} \times -\frac{2r}{\lambda} \times t/t_0$$

$$= -\frac{2t}{t_0} + 4\left(\frac{r}{\lambda}\right)^2 t/t_0$$

Point of inflection is given by

$$-2\frac{t}{t_0} + 4\left(\frac{r}{\lambda}\right)^2 \cdot \frac{t}{t_0} = 0$$

$$\left(\frac{r}{\lambda}\right)^2 = \frac{1}{2}$$

$$\boxed{\frac{r}{\lambda} = \frac{1}{\sqrt{2}} = 0.707}$$

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